SMUG: Towards Robust MRI Reconstruction by Smoothed Unrolling

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Paper

Code

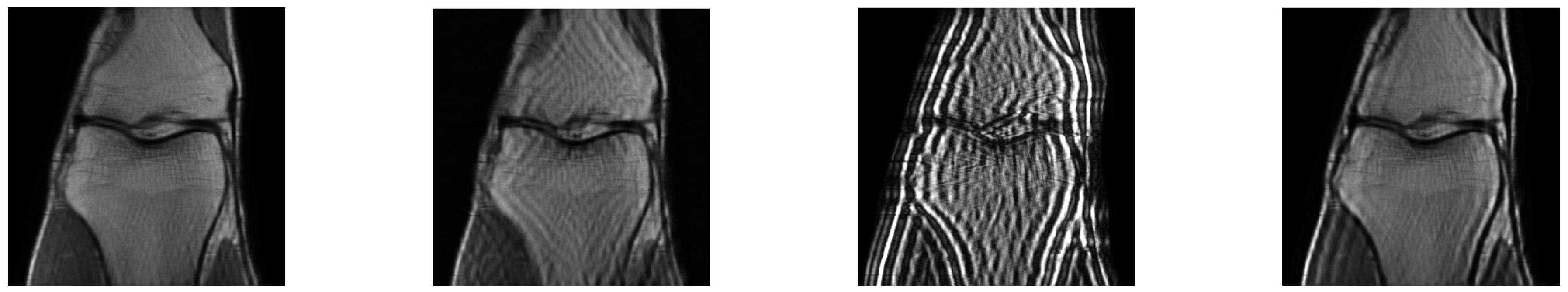
MoDL in MRI Reconstruction

$$\hat{\mathbf{x}}_{\boldsymbol{\theta}} = \arg\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda\|\mathbf{x} - \mathcal{D}_{\boldsymbol{\theta}}(\mathbf{x})\|_2^2$$

- Execute two steps iteratively [1]:
  - (i) Denoising step $\mathbf{z}_n := \mathcal{D}_{\boldsymbol{\theta}}(\mathbf{x}_n)$
  - (ii) Data-consistency step $\mathbf{x}_{n+1} = \arg\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda\|\mathbf{x} - \mathbf{z}_n\|_2^2$

Lack of Robustness in MoDL

- Adversarial input $\underset{\|\boldsymbol{\delta}\|_\infty \le \epsilon}{\text{minimize}} \quad -\|\mathbf{x}_{\text{MoDL}}(\mathbf{A}^H\mathbf{y} + \boldsymbol{\delta}) - \mathbf{t}\|_2^2$
- Change of measurement sampling rate
- Change of number of unrolling steps

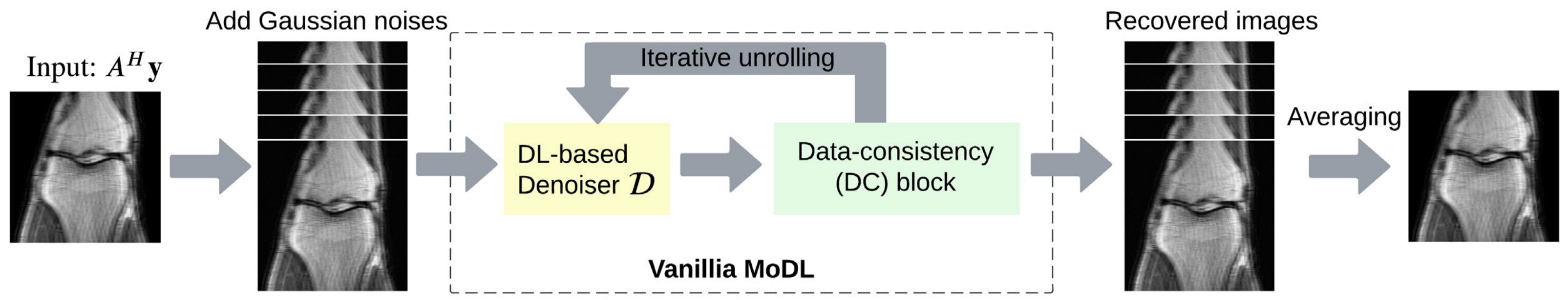


MoDL 25% sampling — With adversarial input — 50% sampling — 2× unrolling steps

Randomized Smoothing (RS)

- RS-E2E [2]: Integrating RS with MoDL in an end-to-end manner

$$g(\mathbf{A}^H\mathbf{y}) = \mathbb{E}_{\boldsymbol{\nu}\sim\mathcal{N}(\mathbf{0},\sigma^2\mathbf{I})}[\mathbf{x}_{\text{MoDL}}(\mathbf{A}^H\mathbf{y} + \boldsymbol{\nu})]$$

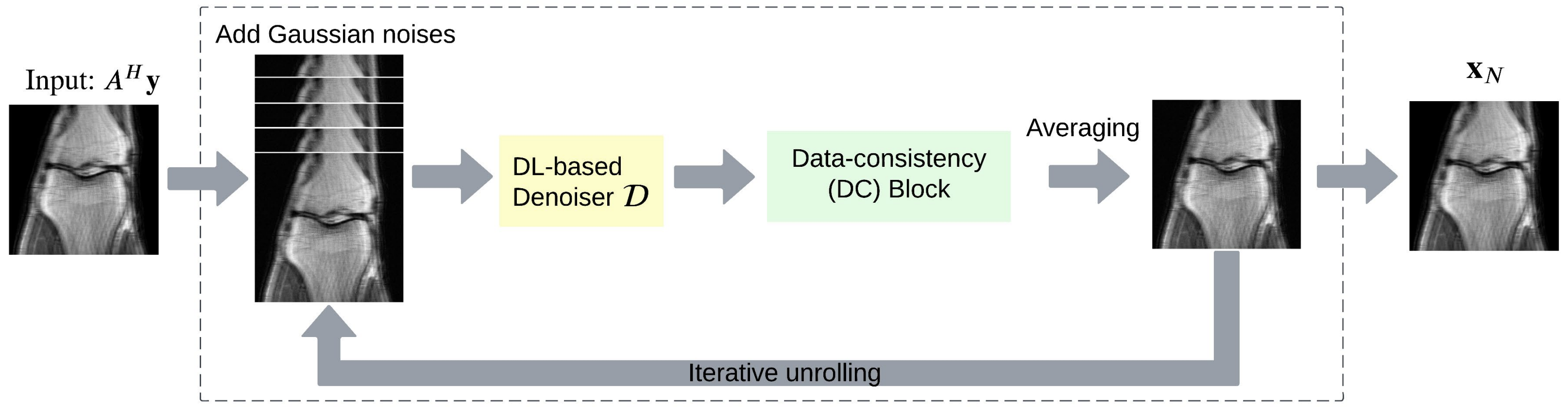


Q1: Where should the RS operator be integrated into MoDL?
Q2: How to design the denoiser in the presence of RS?

SMUG Framework

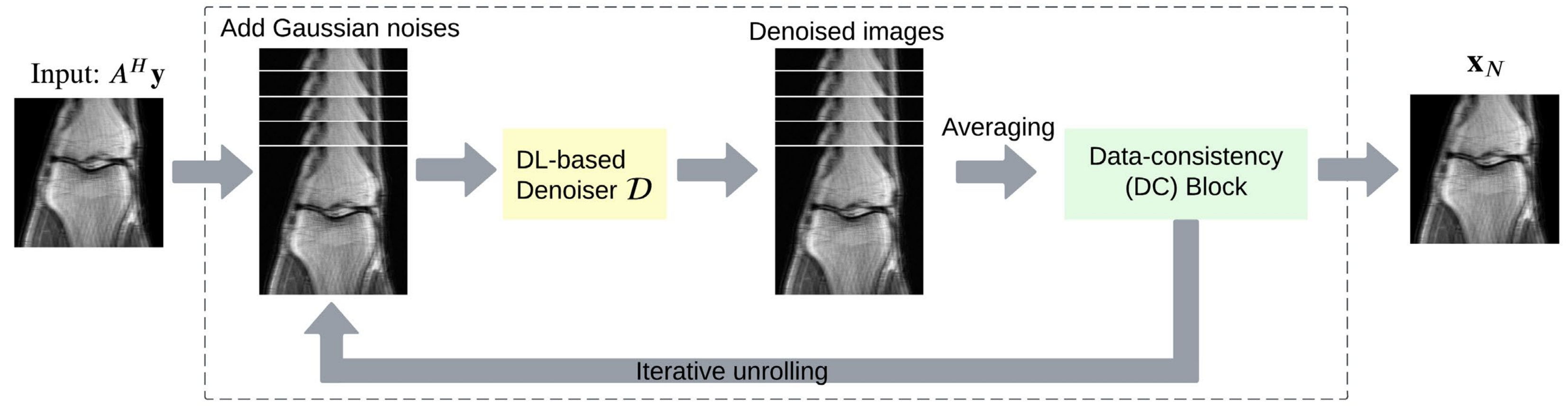
- SMUGv0: RS is incorporated into MoDL at each unrolling step

$$\text{RS}(\mathcal{D} + \mathcal{DC}) = \mathbb{E}_{\boldsymbol{\nu}\sim\mathcal{N}(\mathbf{0},\sigma^2\mathbf{I})}[\mathbf{x}_n(\mathbf{x}_{n-1} + \boldsymbol{\nu})]$$



- SMUG: RS only applies to the denoising network

$$\text{RS}(\mathcal{D}) = \mathbb{E}_{\boldsymbol{\nu}\sim\mathcal{N}(\mathbf{0},\sigma^2\mathbf{I})}[\mathcal{D}_{\boldsymbol{\theta}}(\mathbf{x}_{n-1} + \boldsymbol{\nu})] := \mathbf{z}_n$$



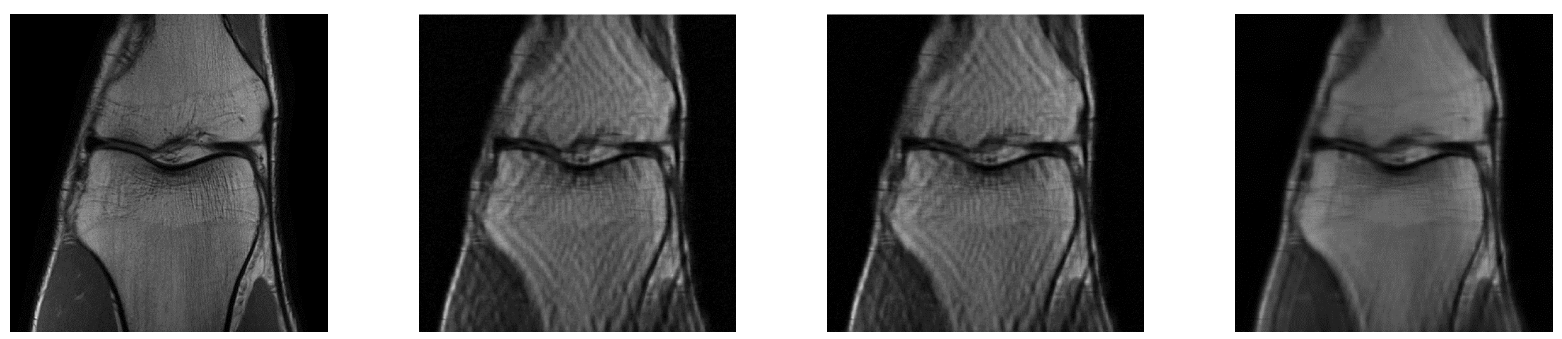
SMUG Training

- Pre-training $\boldsymbol{\theta}_{\text{pre}} = \arg\min_{\boldsymbol{\theta}} \mathbb{E}_{\mathbf{t}\in\mathcal{D}}[\mathbb{E}_{\boldsymbol{\nu}}||\mathcal{D}_{\boldsymbol{\theta}}(\mathbf{t} + \nu) - \mathbf{t}||_2^2]$
  - The denoiser is pre-trained alone to provide a robustness-aware initialization for fine-tuning
- Fine-tuning
  - Unrolled Stability (UStab) loss $\ell_{\text{UStab}}(\boldsymbol{\theta}; \mathbf{y}, \mathbf{t}) = \sum_{n=0}^{N-1} \mathbb{E}_{\boldsymbol{\nu}}||\mathcal{D}_{\boldsymbol{\theta}}(\mathbf{x}_n + \boldsymbol{\nu}) - \mathcal{D}_{\boldsymbol{\theta}}(\mathbf{t})||_2^2$
  - Fine-tuning loss $\ell(\boldsymbol{\theta}; \mathbf{y}, \mathbf{t}) = \lambda_\ell \|\mathbf{x}_N(\boldsymbol{\theta}; \mathbf{A}^H\mathbf{y}) - \mathbf{t}\|_2^2 + \ell_{\text{UStab}}(\boldsymbol{\theta}; \mathbf{y}, \mathbf{t})$

Experiment Results

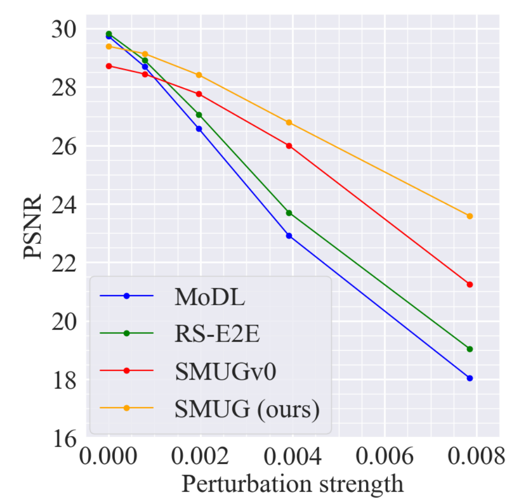
Table 1: Accuracy performance of different methods. 'Clean Accuracy', 'Noise Accuracy', and 'Robust Accuracy' refer to evaluation on benign data, random noise-injected data, and PGD attack-enabled adversarial data, respectively. The relative performance is reported w.r.t vanilla MoDL.

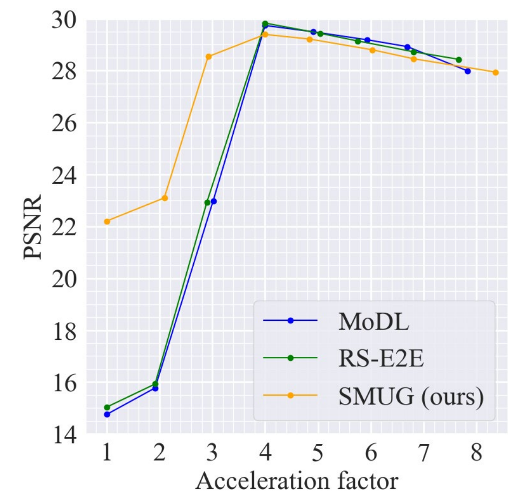
Models	Clean Accuracy		Noise Accuracy		Robust Accuracy	
Metrics	PSNR ↑	SSIM ↑	PSNR ↑	SSIM ↑	PSNR ↑	SSIM ↑
Vanilla MoDL	29.73±3.27	0.900±0.07	28.70±2.77	0.874±0.07	22.91±2.42	0.729±0.07
RS-E2E	**+0.09**±3.24	**+0.002**±0.07	+0.38±2.90	+0.010±0.07	+0.78±2.70	**+0.034**±0.08
SMUGv0	-1.01±3.07	-0.014±0.08	-0.09±2.99	+0.008±0.08	+3.08±2.42	-0.014±0.11
SMUG (ours)	-0.34±3.06	-0.006±0.08	**+0.53**±2.98	**+0.016**±0.08	**+3.87**±2.28	+0.008±0.11



Ground Truth — Vanilla MoDL — RS-E2E — SMUG

Fig. 5: Visualization of ground-truth and reconstructed images using different methods, evaluated on PGD attack-generated adversarial inputs.





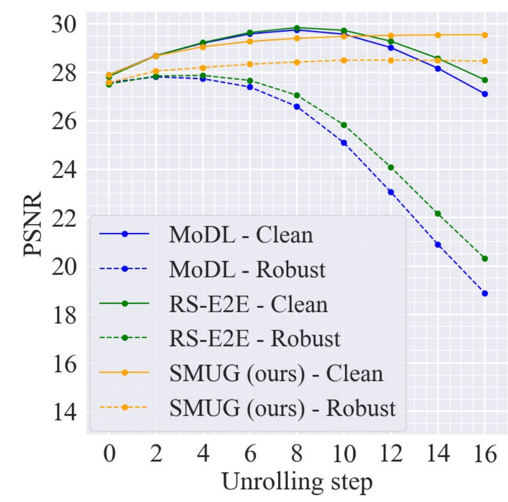


Fig. 6 & 7: PSNR of different methods versus perturbation strength used in PGD-generated adversarial examples (up), measurement sampling rate (4× acceleration i.e. 25% sampling rate) (middle), and number of unrolling step (down).

[1] Hemant K. Aggarwal, Merry P. Mani, and Mathews Jacob, "MoDL: Model-based deep learning architecture for inverse problems," IEEE Trans. Med. Imaging, vol. 38, no. 2, pp. 394–405, Feb. 2019
[2] Adva Wolf, "Making medical image reconstruction adversarially robust," 2019.

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